



Disturbance decoupling by measurement feedback: sensor location

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Received 23 December 2015, accepted 9 March 2016, available online 17 September 2016

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Abstract. The paper addresses the problem on sensor location regarding the solvability of the disturbance decoupling problem by the dynamic measurement feedback (DDDPM). Both, the discrete- and continuous-time nonlinear systems are considered. An exact formula is given to compute a controlled invariant vector function, necessary for the solvability of the DDDPM. Then, two methods are given to find a measured output, which guarantee the solution to the DDDPM. The results are illustrated by several examples.

Key words: nonlinear systems, decoupling, plant reconfiguration, measurement feedback, sensor location.

1. INTRODUCTION

In this paper, we will focus on the problem of the location of the sensors regarding the solvability of the disturbance decoupling problem by the dynamic measurement feedback (DDDPM). Aspects of the sensor location problem have been widely studied in the literature related to different problems. In general, this problem answers the questions how many sensors we need and where they should be placed so that the problem is solvable. Most frequently, one desires to place the sensors so as to be able to estimate the states or system parameters [2,16,18,19] or to detect the changes in the system behaviour [1]. The latter aspect is strongly connected to fault detection and isolation problems [3–7]. The problem considered in this paper falls into the last category. It was shown in [11] that the DDDPM is closely related to the fault tolerant control. In [11], it was assumed that the faults are detected and isolated, after which a subsystem that does not depend explicitly on the faults is found. In this sense, the sensor location problem considered in this paper can be used to ensure that the subsystem one looks for in [11] has the maximal possible dimension.

Note that to choose the number of sensors and their locations such that the DDDPM becomes solvable, one has to be able to check the solvability of the DDDPM. For that reason, this paper relies on the necessary and sufficient solvability conditions for the DDDPM [12]. The conditions given in [12] were developed for discrete-time nonlinear systems and depend on the existence of a certain controlled invariant vector function. To compute this vector function, the algorithms were given in [11] and [12], but neither of them is easily computable.

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In this paper, we first give, under some assumptions, an explicit formula to compute the required controlled invariant vector function. After that the sensor location problem is considered.

A condition is given that guarantees that the selection of an output function H results in the solvability of the DDDPM. One looks for the maximal (in the sense of preorder \leq defined in Section 2) vector function H that satisfies the given condition. The difficulty is that the vector function H one looks for is not unique, and thus many possibilities exist for finding it. We present two methods to find the vector function H , with minimal dimension, which guarantee the solvability of the DDDPM. Note that the measured output H determines the location of the necessary sensors. Which of these two solutions is better depends on the specific situation (cost of sensors, possibility of putting a sensor in such a place etc.) and the system dynamics.

Discrete- and continuous-time nonlinear systems are considered in this paper. The main results are given for the discrete-time case and the differences compared to the continuous-time case are highlighted. Although the proofs of some results are pretty much similar to those in the discrete-time case, there are some results that need a different proof due to the different properties of derivative and shift operators. The preliminary results of this paper were presented in [13] and [14], where the discrete- and continuous-time cases were discussed, respectively.

The paper is organized as follows. Section 2 describes the problem statement and recalls briefly the mathematical tools applied in this paper. Section 3 presents the main results for discrete-time systems. In Section 4, the continuous-time case is discussed and in Section 5 examples are presented. The paper ends with conclusions.

2. PRELIMINARIES

2.1. Problem statement

We give the problem statement for discrete-time systems. The continuous-time case can be stated similarly. In this paper the disturbance decoupling problem under a dynamic measurement feedback (DDPM) is studied for discrete-time nonlinear control systems of the form

$$\begin{aligned} x(k+1) &= f(x(k), u(k), w(k)), \\ y(k) &= h(x(k)), \\ y_*(k) &= h_*(x(k)), \end{aligned} \quad (1)$$

where $x(k) \in X \subseteq \mathbb{R}^n$ is the state, $u(k) \in U \subseteq \mathbb{R}^m$ is the control, $w(k) \in W \subseteq \mathbb{R}^p$ is the unmeasurable disturbance, $y(k) \in Y \subseteq \mathbb{R}^p$ is the measured output, and $y_*(k) \in Y_* \subseteq \mathbb{R}^l$ is the output to be controlled. The DDPM for system (1) can be stated as follows: find a vector function $z(k) = \varphi(x(k))$, $z(k) \in \mathbb{R}^q$ and a regular dynamic measurement feedback of the form

$$\begin{aligned} z(k+1) &= F(z(k), y(k), v(k)), \\ z(0) &= \varphi(x(0)), \\ u(k) &= G(z(k), y(k), v(k)), \end{aligned} \quad (2)$$

where $v(k) \in V \subseteq \mathbb{R}^m$ and $\text{rank}[\partial G/\partial v] = m$, such that the values of the outputs to be controlled $y_*(k)$, for $k \geq 0$, of the closed-loop system (see Fig. 1) are independent of the disturbances $w(k)$. Note that we call the compensator described by (2) regular if it generically defines the (y, z) -dependent one-to-one correspondence between the variables $v(k)$ and $u(k)$. One says that the disturbance decoupling problem is solvable via static output feedback if $u(k) = G(y(k), v(k))$. We are searching for generic solutions, i.e. for solutions that are valid on some open and dense subsets of suitable domains if they are valid on some point of this domain.

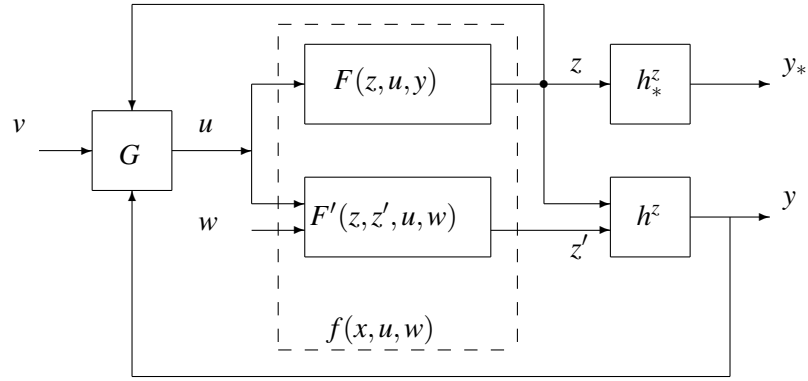


Fig. 1. Closed-loop system, where $(z, z')^T = T(x)$ is a state transformation, $h_*^z(z) := h_*(T^{-1}(z, z'))$ and $h^z(z, z') := h(T^{-1}(z, z'))$.

Note that the solution to the DDDPM depends on the measured output $y(k)$. The main goal of this paper is to find for a given system

$$\begin{aligned} x(k+1) &= f(x(k), u(k), w(k)), \\ y_*(k) &= h_*(x(k)) \end{aligned} \quad (3)$$

a measured output $y(k) = H(x(k))$ such that the DDDPM is solvable for (3).

2.2. The algebra of functions

The mathematical approach called the algebra of functions [20] will be used to address the problem. We recall briefly the definitions and concepts to be used in this paper, see also [15]. Denote by S_D and S_X the sets of vector functions with the domains $D = X \times U \times W$ and X , respectively. On S_D is defined a preorder \leq , which induces an equivalence relation \cong .

Definition 1. (i) Given $\alpha, \beta \in S_D$, one says that $\alpha \leq \beta$ if there exists a function γ such that $\beta(\zeta) = \gamma(\alpha(\zeta))$ for $\zeta \in D$.

(ii) If $\alpha \leq \beta$ and $\beta \leq \alpha$, then α and β are called strictly equivalent, denoted by $\alpha \cong \beta$.

The relation \cong is an equivalence relation and divides the elements of S_D into equivalence classes for which the relation \leq is partial order. The set of these equivalence classes is, together with the relation \leq , a lattice, since $\mathbf{0} := [x, u, w] \leq \alpha \leq \mathbf{1}$ for all $\alpha \in S_D$, where $\mathbf{1}$ is the equivalence class containing constant vector functions. This allows us to define the binary operations \times and \oplus as

$$\alpha \times \beta = \inf(\alpha, \beta), \quad \alpha \oplus \beta = \sup(\alpha, \beta) \quad (4)$$

for all $\alpha, \beta \in S_D$. Note that hereinafter we work with the equivalence classes, i.e. the sign ‘=’ should be understood as ‘ \cong ’.

The previously defined lattice will be connected to the system dynamics (1) through the following binary relation Δ .

Definition 2. Given $\alpha, \beta \in S_X$, one says that $(\alpha, \beta) \in \Delta$ if there exists a function f_* such that for all $(x, u, w) \in D$,

$$\beta(f(x, u, w)) = f_*(\alpha(x), u, w). \quad (5)$$

The binary relation Δ is mostly used for the definition of the operators \mathbf{m} and \mathbf{M} .

Definition 3. (i) The function $\mathbf{m}(\alpha)$ is a minimal vector function $\beta \in S_X$ that satisfies $(\alpha, \beta) \in \Delta$;
(ii) $\mathbf{M}(\beta)$ is a maximal vector function $\alpha \in S_X$ that satisfies $(\alpha, \beta) \in \Delta$.

Some important properties of the relation \leq and operators \oplus , \times , and \mathbf{M} are given by the following Lemma.

Lemma 1 ([20]). Let α , β , and γ be some vector functions from S_X . Then

1. $\alpha \leq \beta \Leftrightarrow \alpha \times \beta = \alpha \Leftrightarrow \alpha \oplus \beta = \beta$;
2. $\alpha \leq \beta \Rightarrow \mathbf{M}(\alpha) \leq \mathbf{M}(\beta)$;
3. $\mathbf{M}(\alpha \times \beta) = \mathbf{M}(\alpha) \times \mathbf{M}(\beta)$.

3. MAIN RESULTS

3.1. The solution of the DDDPM

In this subsection the solution to the DDDPM is given. Compared to [12], improved proofs are provided and an explicit formula is found for the computation of the controlled invariant vector function ξ on which the solution of the DDDPM depends.

First, some important definitions are recalled.

Definition 4. The vector function $\alpha \in S_X$ is said to be (h, f) -invariant if $(\alpha \times h, \alpha) \in \Delta$. In case $h = \mathbf{1}$, the vector function α is said to be f -invariant.

Definition 5. The vector function $\alpha \in S_X$ is said to be controlled invariant if there exists a regular static state feedback $u = G(x, v)$ such that α is f -invariant for the closed-loop system.

Definitions 4 and 5 are generalizations of the concepts of conditioned invariant and controlled invariant distributions, respectively, as given, for instance in [10]. See more about their relationship in [12].

For checking whether a vector function α is (h, f) -invariant (f -invariant), we use the following Lemma.

Lemma 2 [(20)]. The function α is (h, f) -invariant (f -invariant) if and only if

$$\alpha \times h \leq \mathbf{M}(\alpha) \quad (\alpha \leq \mathbf{M}(\alpha)). \quad (6)$$

Note that when vector functions α and β are (h, f) -invariant, then so is $\alpha \times \beta$. This means that when one looks for a minimal (h, f) -invariant vector function α satisfying $\mu \leq \alpha$ for some $\mu \in S_X$, then it is uniquely defined. The same is true for f -invariant vector functions, since these are a special case of (h, f) -invariance.

Lemma 3. Let α and β be minimal (h, f) -invariant and f -invariant functions satisfying the conditions $\mu \leq \alpha$ and $\mu \leq \beta$ for some function μ , respectively. Then $\alpha \leq \beta$.

Proof. Since β is also (h, f) -invariant, then $\alpha \times \beta$ must be (h, f) -invariant, which satisfies $\mu \leq \alpha \times \beta$. Because, by assumption, α is a minimal (h, f) -invariant vector function satisfying the conditions $\mu \leq \alpha$, then $\alpha = \alpha \times \beta$. By Lemma 1, $\alpha \leq \beta$. \square

To present the results on the DDDPM, find first a minimal (containing the maximal number of functionally independent components) vector function $\alpha^0(x)$ such that its forward shift $\alpha^0(f(x, u, w))$ does not depend on the unmeasurable disturbance w . The function $\alpha^0(x)$ plays an important role in the solution of the DDDPM below.

The next theorem gives a condition to check if a system is disturbance decoupled or not.

Theorem 1 [11]. System (1) is disturbance decoupled if and only if there exists an f -invariant function ϕ such that $\alpha^0 \leq \phi \leq h_*$.

Theorem 2. System (1) can be disturbance decoupled by feedback (2) if and only if there exist an controlled invariant function ϕ and an (h, f) -invariant function ψ such that

$$\alpha^0 \leq \psi \leq \phi \leq h_*. \tag{7}$$

Proof. Necessity. Denote by \tilde{f} and $\tilde{\Delta}$ the function f and the relation Δ , respectively, for the closed-loop system (1)–(2). Assume that there exists a feedback (2) that solves the DDDPM. Then, by Theorem 1, there exists an \tilde{f} -invariant function ϕ such that $\alpha^0 \leq \phi \leq h_*$. Obviously, ϕ is controlled invariant for the original system (1). In (2), the function $\varphi =: \psi$ is clearly (h, f) -invariant and the condition $\alpha^0 \leq \psi$ is satisfied. Because ψ is (h, f) -invariant, then $(\psi \times h, \psi) \in \Delta$ and because $u = G(z, y, v)$ depends only on $z = \psi(x)$ and $y = h(x)$, then $(\psi \times h, \psi) \in \tilde{\Delta}$ holds, i.e. ψ is (h, \tilde{f}) -invariant. Then $\psi \leq \phi$ by Lemma 3.

Sufficiency. Given in [12]. □

The algorithm below can be used to find the vector function ψ in Theorem 2. It computes the minimal (h, f) -invariant vector function α that satisfies the condition $\alpha^0 \leq \alpha$.

Algorithm 1 [15]. Given α^0 , compute recursively for $i \geq 1$, using the formula

$$\alpha^{i+1} = \alpha^i \oplus \mathbf{m}(\alpha^i \times h), \tag{8}$$

the sequence of non-decreasing vector functions $\alpha^0 \leq \alpha^1 \leq \alpha^2 \leq \dots \leq \alpha^i \leq \dots$. The sequence converges in a finite number of steps, since if $\alpha^i \neq \alpha^{i-1}$, the number of components of the function α^i is less than that of the function α^{i-1} , $i = 1, 2, \dots$. This means that there exists a finite j such that $\alpha^j \neq \alpha^{j-1}$ but $\alpha^{j+l} = \alpha^j$ for all $l \geq 1$. Define $\alpha := \alpha^j$.

Since α is the minimal (h, f) -invariant function satisfying the condition $\alpha^0 \leq \alpha$, this function is the best choice for ψ in Theorem 2.

Finding the controlled invariant vector function ϕ in Theorem 2 is more complicated. Below, in Theorem 3 a formula for computing ϕ is given under some assumptions.

Let $h_* = [h_{*1}, \dots, h_{*L}]^T$ and denote by r_i and d_i the relative degrees of the function $h_{*i}(x)$ with respect to the control input u and the disturbance w , respectively. Moreover, we use the notations $y_{*i}(k) = h_{*i}(x(k)) =: h_{*i,1}(x(k)), \dots, y_{*i}(k + r_i - 1) =: h_{*i,r_i}(x(k)), y_{*i}(k + r_i) =: h_{*i,r_i+1}(x(k), u(k))$. We make the following assumptions:

Assumption 1. $d_i > r_i$.

From the definition of r_i , one has $h_{*i,r_i+1} = \hat{f}_i(x, u)$ for some \hat{f}_i .

Definition 6 (vector relative degree).¹ The vector (r_1, \dots, r_L) is called a vector relative degree of output y_* if

$$\text{rank} \left[\frac{\partial (\hat{f}_1(x, u), \dots, \hat{f}_L(x, u))^T}{\partial u} \right] = L$$

generically, i.e. everywhere except on the set of zero measure.

Assumption 2. The output y_* has a vector relative degree.

Note that Assumption 1 is a standard assumption made in the solution of the disturbance decoupling problem even when the solution is looked for in the form of state feedback. Assumption 2 may be, in principle, replaced by the assumption of right invertibility (regarding the output y_*). However, this type of assumption is often made for the same of simplification; in particular here it allows finding the explicit formula to compute the function ϕ in Theorem 2, allowing us to make the result of Theorem 2 constructive.

Consider the set of equations

$$\hat{f}_i(x, u) = v_i \quad i = 1, \dots, L. \tag{9}$$

¹ This definition is in accordance with Remark 5.1.3. of [9].

Under Assumptions 1 and 2, the set of equations (9) is generically solvable for u .

Note that the definition of the operator \mathbf{M} depends on the function f in Eq. (1). By $\tilde{\mathbf{M}}$ we denote the operator \mathbf{M} defined by the function \tilde{f} , which is the state transition map for the closed-loop system (1)–(2).

Theorem 3. *Under Assumptions 1 and 2, the maximal controlled invariant function ξ that satisfies the inequality $\xi \leq h_*$ may be computed by the formula*

$$\xi := \prod_{i=1}^L (h_{*i,1} \times \cdots \times h_{*i,r_i}). \quad (10)$$

Proof. Since the set of equations (9) is solvable for u , one can find a static state feedback by solving these equations. We show that the function ξ in (10) is \tilde{f} -invariant. By the third property of Lemma 1, one obtains

$$\tilde{\mathbf{M}}(\xi) = \prod_{i=1}^L (\tilde{\mathbf{M}}(h_{*i,1}) \times \cdots \times \tilde{\mathbf{M}}(h_{*i,r_i})),$$

and from the definition of the operator $\tilde{\mathbf{M}}$, one has $\tilde{\mathbf{M}}(h_{*i,j}) = h_{*i,j+1}$, $j = 1, \dots, r_i - 1$. Since by (9) $h_{*i,r_i+1} = v_i$, $\tilde{\mathbf{M}}(h_{*i,r_i}) = \mathbf{1}$. Therefore,

$$\begin{aligned} \tilde{\mathbf{M}}(\xi) &= \prod_{i=1}^L (h_{*i,2} \times h_{*i,3} \times \cdots \times h_{*i,r_i} \times \mathbf{1}) \\ &\geq \prod_{i=1}^L (h_{*i,1} \times h_{*i,2} \times \cdots \times h_{*i,r_i} \times \mathbf{1}) = \xi, \end{aligned}$$

i.e. $\xi \leq \tilde{\mathbf{M}}(\xi)$. By Lemma 2 the function ξ is an \tilde{f} -invariant or controlled invariant function for the original system.

Next, let β be another controlled invariant function such that $\beta \leq h_* = \prod_{i=1}^L h_{*i,1}$. As β is controlled invariant, then $\beta \leq \tilde{\mathbf{M}}(\beta)$. Because $\beta \leq h_* \Rightarrow \tilde{\mathbf{M}}(\beta) \leq \tilde{\mathbf{M}}(h_*)$, see [15], one obtains

$$\beta \leq \tilde{\mathbf{M}}(\beta) \leq \tilde{\mathbf{M}}(h_*) = \tilde{\mathbf{M}}\left(\prod_{i=1}^L h_{*i,1}\right) = \prod_{i=1}^L \tilde{\mathbf{M}}(h_{*i,1}) = \prod_{i=1}^L h_{*i,2}.$$

By analogy, $\beta \leq \prod_{i=1}^L h_{*i,j}$ for $j = 3, \dots, r_i$. Then, by the definition of operation \times , $\beta \leq \prod_{i=1}^L (h_{*i,1} \times h_{*i,2} \times \cdots \times h_{*i,r_i}) = \xi$, which means that ξ is the maximal \tilde{f} -invariant function satisfying the condition $\xi \leq h_*$. \square

Since ξ is the maximal controlled invariant function satisfying the inequality $\xi \leq h_*$, this function is the best choice for ϕ in Theorem 2.

3.2. Sensor location

In this subsection, two methods for finding the unknown measured output function $H(x)$ that make the DDDPM solvable are described. The goal is to find the vector function H as maximal² as possible because typically one wants a minimal number of sensors to be used. By Theorem 2, the function H must guarantee the existence of an (H, f) -invariant vector function ψ and a controlled invariant vector function ϕ , satisfying $\alpha^0 \leq \psi \leq \phi \leq h_*$. Note that the controlled invariant vector function ϕ does not depend on the choice of $H(x)$, and thus it can be taken equal to the vector function ξ (which has to satisfy $\alpha^0 \leq \xi$), defined by

² Maximal in terms of the preorder \leq .

(10). In Case 1 below, the function H is computed based on the function α^0 , in Case 2, the function H is computed based on the function ξ .

Case 1. In this case, the measured output H is chosen such that the (H, f) -invariant vector function ψ can be taken $\psi = \alpha^0$. Note that by Algorithm 1, we have $\alpha^1 = \alpha^0 \oplus \mathbf{m}(\alpha^0 \times H)$. If the choice H guarantees that $\mathbf{m}(\alpha^0 \times H) \leq \alpha^0$, then $\alpha^1 = \alpha^0$, $\alpha = \alpha^0$ and one can take $\psi = \alpha$. Therefore $\psi \leq \xi$ and the DDDPM is solvable for the given H . The condition $\mathbf{m}(\alpha^0 \times H) \leq \alpha^0$ is equivalent to the condition $\alpha^0 \times H \leq \mathbf{M}(\alpha^0)$, which is easier for computing the maximal H satisfying the last inequality.

Case 2. In this case, the measured output H is chosen such that the (H, f) -invariant vector function ψ can be taken $\psi = \xi$. By definition, one has to find the maximal function H such that $\xi \times H \leq \mathbf{M}(\xi)$ is valid, meaning the function ξ is (H, f) -invariant. Since the condition $\alpha^0 \leq \xi$ holds, the function ξ can be taken as ψ .

In general, one may take any vector function η that satisfies $\alpha^0 \leq \eta \leq \xi$ and find the maximal function H such that $\eta \times H \leq \mathbf{M}(\eta)$ is valid. In this case ψ can be taken equal to η .

In all the cases described above, one needs to find the maximal vector function H that satisfies the condition $\lambda \times H \leq \mathbf{M}(\lambda)$ for the given λ . Loosely speaking, the function H ‘helps’ the function λ to satisfy the condition $\lambda \times H \leq \mathbf{M}(\lambda)$. Note that the maximal vector function H that satisfies the condition $\lambda \times H \leq \mathbf{M}(\lambda)$ is not unique and that there are many different ways to find H .

It is easy to see that choosing $H = \mathbf{M}(\lambda)$ guarantees always that $\lambda \times H \leq \mathbf{M}(\lambda)$. However, this choice is not in general maximal³. The previous choice is maximal if and only if $\mathbf{M}(\lambda) \oplus \lambda = \mathbf{1}$. If not, then one has to eliminate the elements of $\mathbf{M}(\lambda) \oplus \lambda$ from $\mathbf{M}(\lambda)$, i.e. to find μ such that

$$\mathbf{M}(\lambda) = \mu \times [\mathbf{M}(\lambda) \oplus \lambda]. \tag{11}$$

Then, the choice $H = \mu$ guarantees that $\lambda \times H \leq \mathbf{M}(\lambda)$. Really, now one has $\mathbf{M}(\lambda) = H \times [\mathbf{M}(\lambda) \oplus \lambda]$, and since $\mathbf{M}(\lambda) \oplus \lambda$ can be written, by the definition of \oplus , in terms of λ , $\mathbf{M}(\lambda)$ can be written in terms of H and λ , i.e. $\lambda \times H \leq \mathbf{M}(\lambda)$. The maximality of H is guaranteed if the dimension of the vector μ in (11) also satisfies

$$\dim \mu = \dim \mathbf{M}(\lambda) - \dim [\mathbf{M}(\lambda) \oplus \lambda].$$

This assures that there are no elements in vector μ that can be written in terms of λ and the other elements of μ .

Another, more heuristic way of finding the maximal vector function H that satisfies $\lambda \times H \leq \mathbf{M}(\lambda)$ is described below. Take H such that its elements are

- (i) all the functions $\mathbf{M}(\lambda)_i$ that are components of the vector function $\mathbf{M}(\lambda)$ and satisfy the condition $\lambda \not\leq \mathbf{M}(\lambda)_i$;
- (ii) all the variables x_i on which the vector function $\mathbf{M}(\lambda)$ depends, but the vector function λ does not.

This H is, in general, not maximal but it can be simplified as follows:

- (iii) one can replace H by the equivalent but ‘simpler’ vector function H' ;
- (iv) some of the elements of H' , that can be written in terms of λ and the other elements of H' , can be removed.

Example 1 (Demonstration of the points (iii) and (iv)). Let $\lambda(x) = [x_1 + x_2, x_2 + x_3]^T$ and $\mu(x) = [x_1 + x_3, \ln(x_2) + x_4]^T$. Then by the procedure above, one gets $H(x) = [x_1 + x_3, \ln(x_2) + x_4, x_4]^T$. It is easy to see that $H(x)$ is equivalent to $H'(x) = [x_1 + x_3, \ln(x_2), x_4]^T$. Now, $\ln(x_2)$ can be removed since it can be written in terms of λ and the other elements of H' : $\ln(x_2) = \ln(0.5(\lambda_1(x) + \lambda_2(x) - H'_1(x)))$. Thus one gets $H''(x) = [x_1 + x_3, x_4]^T$.

Example 2 (Demonstration of the heuristic way to improve the solution). Let $\lambda(x) = [x_1 + x_2, \sin(x_2) + x_3]^T$ and $\mu(x) = [x_1 + x_3, x_4]^T$. Then one gets $H(x) = [x_1 + x_3, x_4]^T$ according to (i) and (ii) of the procedure above. This function can not be simplified. However, by adding x_2 to H as the third component, we can simplify it

³ In terms of the preorder \leq .

since $H_1(x) = x_1 + x_3 = \lambda_1(x) + \lambda_2(x) - H_3(x) - \sin(H_3(x))$. So, we get $H'(x) = [x_2, x_4]^T$ as a solution. To resume, consider the variables x_j such that λ depends on x_j but μ does not, and add these variables to H .

To resume, the following procedure is suggested to solve the sensor location problem under consideration.

Step 1. Find the functions α^0 and ξ .

Step 2. Find the output functions H_1 and H_2 solving the inequalities $\alpha^0 \times H_1 \leq \mathbf{M}(\alpha^0)$ and $\xi \times H_2 \leq \mathbf{M}(\xi)$, respectively.

Step 3. Choose the better solution based on the cost of sensors in respective places or possibilities of putting the sensor in such a place.

There is also a possibility that the initial system has some sensors described by the output function $y = h(x)$, but they are deficient to solve the DDDPM. In this case the problem is to find the additional sensors, described by the output function $y_a = H_a(x)$, such that the DDDPM becomes solvable for the output (y, y_a) . This problem can be solved by constructing the function H as shown above and finding the solution of the inequality $h \times H_a \leq H$ by the methods developed above.

4. CONTINUOUS-TIME CASE

In this section continuous-time systems are considered. It is shown that results similar to those in the discrete-time case are valid for the solvability of the DDDPM and the sensor location problem. Therefore, only the most important differences are discussed in this section.

The problem statement of the DDDPM is similar for continuous-time systems except that the systems are in the form

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), w(t)), \\ y(t) &= h(x(t)), \\ y_*(t) &= h_*(x(t)), \end{aligned} \quad (12)$$

where $x(t) \in X \subseteq \mathbb{R}^n$, $u(t) \in U \subseteq \mathbb{R}^m$, $w(t) \in W \subseteq \mathbb{R}^p$, $y(t) \in Y \subseteq \mathbb{R}^p$, $y_*(t) \in Y_* \subseteq \mathbb{R}^L$ and one searches for the regular feedback

$$\begin{aligned} \dot{z}(t) &= F(z(t), y(t), v(t)), \\ z(0) &= \varphi(x(0)), \\ u(t) &= G(z(t), y(t), v(t)), \end{aligned} \quad (13)$$

where $z(t) \in \mathbb{R}^q$.

In the methodology, the main difference between the discrete- and continuous-time cases is the definition of the relation Δ . In the continuous-time case the binary relation Δ is defined as follows.

Definition 7. Given $\alpha, \beta \in S_X$, one says that $(\alpha, \beta) \in \Delta$ if there exists a function f_* such that for all $(x, u, w) \in D$,

$$\dot{\beta} = f_*(\alpha(x), u, w). \quad (14)$$

Note that here one has to assume that the vector function β is differentiable. This makes Definition 7 more restrictive than Definition 2, where β can also be non-smooth.

Of course, since the binary relation Δ is defined differently in discrete- and continuous-time cases, the properties of the operators \mathbf{m} and \mathbf{M} are slightly different in the continuous-time case. Namely, property 2 in Lemma 1 is different, taking the form

$$\alpha \leq \beta \Rightarrow \alpha \times \mathbf{M}(\alpha) \leq \mathbf{M}(\beta). \quad (15)$$

Now, one can generalize Theorems 1 and 2 to the continuous-time case.

Theorem 4. System (12) is disturbance decoupled if and only if there exists an f -invariant function ϕ such that $\alpha^0 \leq \phi \leq h_*$.

Proof. Coincides with that in the discrete-time case given in [11]. \square

Theorem 5. System (12) can be disturbance decoupled by feedback (13) if and only if there exist a controlled invariant function ϕ and an (h, f) -invariant function ψ such that

$$\alpha^0 \leq \psi \leq \phi \leq h_*. \tag{16}$$

Proof. Necessity. The proof coincides with the proof of Theorem 2.

Sufficiency. The proof differs from the discrete-time case due to different properties of the operator \mathbf{M} . Since ϕ is controlled invariant, there exists a static state feedback $u = G(x, v)$ such that $\dot{\phi} = \frac{\partial \phi}{\partial x} f(x, u, w) = \chi(\phi(x), v)$ for some function χ . Since $\alpha^0 \leq \phi \leq h_*$, then by Theorem 4, the closed-loop system is disturbance decoupled. It remains to show that function G depends only on the variables z, y , and v . Take in (13) $z(t) = \phi(x(t))$ and set in (16) $\psi := \phi$. Since $\phi \leq \phi$, then $\phi \times \mathbf{M}(\phi) \leq \mathbf{M}(\phi)$ by (15). It follows from the definitions of the (h, f) -invariant function that $\phi \times h \leq \mathbf{M}(\phi)$; since $\phi \leq \phi$, then by the definition of the operation \times , $\phi \times h \leq \phi \times \mathbf{M}(\phi)$. Therefore, one gets $\phi \times h \leq \mathbf{M}(\phi)$. By the definition of the operator \mathbf{M} , $\mathbf{M}(\phi) \times u \leq \frac{\partial \phi}{\partial x} f(x, u, w) = \dot{\phi}$ (one takes into account that $\frac{\partial \phi}{\partial x} f(x, u, w)$ does not depend on w since $\alpha^0 \leq \phi$). Thus $\phi \times h \times u \leq \mathbf{M}(\phi) \times u \leq \dot{\phi} = \chi(\phi(x), v)$. This means that χ can be written in terms of z, y , and v and then the function G depends also only on z, y , and v . \square

Just like above, one can use Algorithm 1 to compute the minimal (h, f) -invariant vector function α satisfying $\alpha^0 \leq \alpha$. However, here we give another possibility for computing α since unlike in the discrete-time case, there does not exist a formula or an algorithm to compute $\mathbf{m}(\beta)$ for a given β . Observe that here unlike in Algorithm 1 we do not rely on the operator \mathbf{m} but on the operator \mathbf{M} .

Given α^0 , compute recursively for $i \geq 0$, using the formula

$$\frac{\partial \alpha^{i+1}(x)}{\partial t} = \frac{\partial \alpha^i(x)}{\partial t} \oplus (\alpha^i(x) \times h(x) \times u), \tag{17}$$

(we will use below the notation $\dot{\alpha}^{i+1} = \dot{\alpha}^i \oplus (\alpha^i \times h \times u)$ for simplicity) the sequence of non-decreasing functions

$$\alpha^0 \leq \alpha^1 \leq \alpha^2 \leq \dots \leq \alpha^i \leq \dots$$

(follows from the property of the operation \oplus). The sequence converges in a finite number of steps since if $\alpha^i \neq \alpha^{i-1}$, the number of components of the function α^i is less than that of the function α^{i-1} or equal to it, $i = 1, 2, \dots$. This means that there exists a finite j such that $\alpha^j \neq \alpha^{j-1}$ but $\alpha^{j+l} = \alpha^j$ for all $l \geq 1$. Define $\alpha := \alpha^j$.

Note that finding $\alpha^{(i+1)}$ from its time derivative requires that the following set of partial differential equations be solved for α^{i+1} :

$$\frac{\partial \alpha^{i+1}}{\partial x} f(x, u, w) = \dot{\alpha}^{i+1},$$

where on the right-hand side is the vector function obtained from formula (17).

Theorem 6. The function α is minimal (h, f) -invariant satisfying the condition $\alpha^0 \leq \alpha$.

Proof. Since $\alpha := \alpha^j = \alpha^{j+l}$, then $\dot{\alpha}^j = \dot{\alpha}^{j+l}$ and $\dot{\alpha}^{j+1} = \dot{\alpha}^j = \dot{\alpha}^j \oplus (\alpha^j \times h \times u)$. This equality implies the relation $(\alpha^j \times h \times u) \leq \dot{\alpha}^j$ or $(\alpha \times h \times u) \leq \dot{\alpha}$. The last inequality can be rewritten in the form $(\alpha \times h \times u) \leq \frac{\partial \alpha}{\partial x} f(x, u, w)$, which means that $(\alpha \times h, \alpha) \in \Delta$, i.e. α is an (h, f) -invariant function. Assume that β is another (h, f) -invariant function satisfying the condition $\alpha^0 \leq \beta$. Then $(\beta \times h \times u) \leq \frac{\partial \beta}{\partial x} f(x, u, w) = \dot{\beta}$; this relation and the condition $\alpha^0 \leq \beta$ imply that $(\alpha^0 \times h \times u) \leq \dot{\beta}$ and $\dot{\alpha}^0 \leq \dot{\beta}$. Since $\dot{\alpha}^1 = \dot{\alpha}^0 \oplus (\alpha^0 \times h \times u)$, then by the definition of the operation \oplus one obtains $\dot{\alpha}^1 \leq \dot{\beta}$; since the function f is surjective, $\alpha^1 \leq \beta$. By analogy, $\alpha^i \leq \beta$ for $i \geq 1$, so $\alpha \leq \beta$. \square

Next, it is shown that Theorem 3 can also be generalized for continuous-time systems. For that, let $y_{*i}(t) = h_{*i}(x(t)) =: h_{*i,1}(x(t))$, $y_{*i}^{(1)}(t) =: h_{*i,2}(x(t))$, $y_{*i}^{(r_i-1)}(t) =: h_{*i,r_i}(x(t))$ and $h_{*i,r_i+1}(x(t), u(t)) := y_{*i}^{(r_i)}(t) = \widehat{f}_i(x(t), u(t))$ for some \widehat{f}_i .

Theorem 7. Under Assumptions 1 and 2, the maximal controlled invariant function ξ that satisfies the inequality $\xi \leq h_*$ may be computed by formula (10).

Proof. The proof consists of two parts. The first part (ξ computed from (10) is \widetilde{f} -invariant) practically coincides with the respective part of Theorem 3. In the second part of the proof (ξ is maximal among such invariant functions) there is a small difference due to the different properties of the operator \mathbf{M} in the continuous-time case. In particular, if $\beta \leq h_*$, then $\beta \times \widetilde{\mathbf{M}}(\beta) \leq \widetilde{\mathbf{M}}(h_*)$, but since $\beta \leq \widetilde{\mathbf{M}}(\beta)$, one obtains $\mathbf{M}(\beta) \leq \mathbf{M}(h_*)$, which agrees with the discrete-time case. \square

As above, the functions α and ξ are the best choices for ψ and ϕ , respectively, in Theorem 5.

Recall that in Section 3 we described methods for finding the unknown measured output function $H(x)$ that makes the DDDPM solvable. In the continuous-time case the methods are the same. That is, one wants to make a vector function λ that satisfies $\alpha^0 \leq \lambda \leq \xi$, (H, f) -invariant. The methods for computing such H are the same for discrete- and continuous-time cases.

5. EXAMPLES

Example 3. Consider the control system

$$\begin{aligned}
 x_1(k+1) &= \vartheta_1 x_1(k)^2 \text{sign}(x_1(k)) + \vartheta_2 x_2(k) \\
 &\quad + x_1(k) + \vartheta_6 u_1(k) + \vartheta_6 u_2(k), \\
 x_2(k+1) &= \vartheta_3 x_1(k) x_2(k) + x_2(k) + \frac{\vartheta_7}{x_1(k)} u_1(k) \\
 &\quad + x_3(k) + \vartheta_9 u_3(k), \\
 x_3(k+1) &= \vartheta_4 x_4(k) + \vartheta_5 x_3(k) \text{sign}(x_1(k)) \\
 &\quad + x_3(k) + \vartheta_9 u_3(k), \\
 x_4(k+1) &= \vartheta_{10} x_3(k) + x_4(k), \\
 x_5(k+1) &= \vartheta_{11} x_1(k) x_2(k) + x_5(k), \\
 y_*(k) &= x_4(k).
 \end{aligned} \tag{18}$$

Equations (18) constitute a simplified sampled-data model of the underwater vehicle moving on a vertical plane developed under the assumptions of small x_1 and x_2 values, see [17]. Model variables have the following meaning: x_1 is the velocity; x_2 is the angle of the trajectory; x_4 and x_3 are the trim and its time derivative, respectively; x_5 is the depth. The model coefficients $\vartheta_1 \div \vartheta_{11}$ characterize the masses, inertia, and the structural features of the vehicle. The inputs u_1 , u_2 , and u_3 are the forces of the upper and bottom stern thrusters and the vertical bow thruster, respectively.

Our goal is to find the measurement output in such a manner that allows us to solve the DDDPM.

Compute, according to (10), the vector function $\xi = [x_4, \vartheta_{10} x_3 + x_4]^T$, which is equivalent to $\xi = [x_3, x_4]^T$. Then $\mathbf{M}(\xi) = [\vartheta_{10} x_3 + x_4, x_3 \vartheta_4 x_4 + \vartheta_5 x_3 \text{sign}(x_1)]^T$. Note that $\alpha^0 = [x_1, x_3, x_4, x_5]^T$ and the necessary condition $\alpha^0 \leq \xi$ is satisfied. Since $\mathbf{M}(\xi) \oplus \xi = \vartheta_{10} x_3 + x_4$, then $\dim H = 1$. It is obvious that the inequality $\xi \times H \leq \mathbf{M}(\xi)$ is valid for $H(x) = x_1$.

Another possibility is to define H such that the vector function α^0 becomes (H, f) -invariant. For that one can take $H = x_2$, since $\alpha^0 \times H = \mathbf{0}$ and thus $\alpha^0 \times H \leq \mathbf{M}(\alpha^0)$ is satisfied.

Example 4. Consider the control system described by the equations

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -\sqrt{x_1(t)} + \sqrt{x_3(t)} + \gamma_a u_1(t), \\ -\sqrt{x_2(t)} + \sqrt{x_4(t)} + \gamma_b u_2(t), \\ -\sqrt{x_3(t)} + (1 - \gamma_b)u_2(t) + w(t), \\ -\sqrt{x_4(t)} + (1 - \gamma_a)u_1(t) + w(t), \end{bmatrix} \\ y_{*1}(t) &= x_1(t), \\ y_{*2}(t) &= x_2(t). \end{aligned}$$

These equations constitute the normalized four-bank benchmark process [8], where the state variables x_1 , x_2 , x_3 , and x_4 are liquid levels of Tank 1, Tank 2, Tank 3, and Tank 4, respectively; u_1 and u_2 are the control inputs.

Compute the vector functions $\alpha^0(x) = [x_1, x_2, x_3 - x_4]^T$ and $\xi(x) = [x_1, x_2]^T$.

First, find H such that ξ becomes (H, f) -invariant. For that, compute $\mathbf{M}(\xi)(x) = [-\sqrt{x_1} + \sqrt{x_3}, -\sqrt{x_2} + \sqrt{x_4}]^T$. Since $\mathbf{M}(\xi) \oplus \xi = \mathbf{1}$, then $\dim H = 2$. The inequality $\xi \times H \leq \mathbf{M}(\xi)$ is valid if $H(x) = [x_3, x_4]^T$.

Next, find H such that α^0 becomes (H, f) -invariant. Since $\alpha^0 \times H = [x_1, x_2, x_3, x_4]^T = \mathbf{0}$ for $H = x_3$ or $H = x_4$, then $\alpha^0 \times H \leq \mathbf{M}(\alpha^0)$ for both cases. The choice is $y = H(x) = x_3$.

Set $z := x_3 - x_4$, the compensator is described by the equations

$$\begin{aligned} \dot{z}(t) &= -\sqrt{z(t)} + \sqrt{y(t) - z(t)} \\ &\quad + (1 - \gamma_b)u_2(t) - (1 - \gamma_a)u_1(t), \\ u_1(t) &= (1/\gamma_a)(v_1(t) - \sqrt{y(t)}), \\ u_2(t) &= (1/\gamma_b)(v_2(t) - \sqrt{y(t) - z(t)}). \end{aligned}$$

Example 5. Consider the control system described by the equations

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} \sin(u_1(t) - x_1(t) + x_5(t)), \\ u_2(t) - w(t)x_2^2(t), \\ w(t)x_2(t), \\ x_1(t) \cos(x_3(t)) + u_2(t), \\ x_1(t)x_4(t), \end{bmatrix} \\ y_{*1}(t) &= x_1(t), \\ y_{*2}(t) &= x_5(t). \end{aligned}$$

Compute $\alpha^0(x) = [x_1, (\ln(x_2) + x_3), x_4, x_5]^T$ and $\xi(x) = [x_1, x_4, x_5]^T$ by (10). Since $r_1 = 1$, $r_2 = 2$, $d_1 = 4$, and $d_2 = 3$, Assumption 1 holds. Equations (9) are of the form

$$\begin{aligned} \sin(u_1 - x_1 + x_5) &= v_1, \\ (\cos(x_3) + u_2) \sin(u_1 - x_1 + x_5) &= v_2. \end{aligned} \tag{19}$$

Clearly, Assumption 2 holds and these equations are solvable for u_1 and u_2 .

Observe that sometimes one may simplify the solution if in (9) $\widehat{f}_i(x(t), u(t))$ may be rewritten in the form of a composite function for some ψ and \bar{f}_i :

$$\widehat{f}_i(x(t), u(t)) = \psi(\bar{f}_i(x(t), u(t))).$$

For our example, one has $y_{*1}^{(1)}(t) = \sin(u_1(t) - x_1(t) + x_5(t))$, then $\psi(\cdot) = \sin(\cdot)$ and $\bar{f}_1(x, u) = u_1 - x_1 + x_5$. As a result, equations (19) take the form

$$\begin{aligned} u_1 - x_1 + x_5 &= v_1, \\ (\cos(x_3) + u_2) \sin(v_1) &= v_2. \end{aligned}$$

According to Case 1 in Section 3.2, $\mathbf{M}(\alpha^0)(x) = [x_1 - x_5, x_2, x_1 \cos(x_3), x_1 x_4]^T$. Clearly, one has to set $y = H(x) = x_2$ to satisfy the condition $\alpha^0 \times H \leq \mathbf{M}(\alpha^0)$.

The compensator corresponding to this solution is given by

$$\begin{aligned} \dot{z}_1(t) &= \sin(v_1(t)), \\ \dot{z}_2(t) &= \frac{1}{y(t)} \left(\frac{v_2(t)}{\sin(v_1(t))} - \cos(z_2(t) - \ln(y(t))) \right), \\ \dot{z}_3(t) &= z_1(t)v_2(t)/\sin(v_1(t)), \\ \dot{z}_4(t) &= z_1(t)z_3(t), \\ u_1(t) &= v_1(t) + z_1(t) - z_4(t), \\ u_2(t) &= \frac{v_2(t)}{\sin(v_1(t))} - \cos(z_2(t) - \ln(y(t))). \end{aligned}$$

Here $z := \alpha^0(x)$, i.e. $z_1 := x_1$, $z_2 := \ln(x_2) + x_3$, $z_3 := x_4$, $z_4 := x_5$.

According to Case 2 in Section 3.2, $\mathbf{M}(\xi)(x) = (x_1 - x_5) \times x_1 \cos(x_3) \times x_1 x_4$. Clearly, one has to set $y = H(x) = x_3$ to satisfy the condition $\xi \times H \leq \mathbf{M}(\xi)$.

The compensator corresponding to this solution is described by the equations

$$\begin{aligned} \dot{z}'_1(t) &= \sin(v_1(t)), \\ \dot{z}'_2(t) &= z'_1(t)v_2(t)/\sin(v_1(t)), \\ \dot{z}'_3(t) &= z'_1(t)z'_2(t), \\ u_1(t) &= v_1(t) + z'_1(t) - z'_3(t), \\ u_2(t) &= \frac{v_2(t)}{\sin(v_1(t))} - \cos(y(t)), \end{aligned}$$

where $z' := \xi(x)$, i.e. $z'_1 := x_1$, $z'_2 := x_4$, $z'_3 := x_5$.

6. CONCLUSION

In this paper, the DDDPM was addressed for discrete- and continuous-time nonlinear systems. A formula was given to find, under some assumptions, a controlled invariant function ξ , which is essential in the solution of the DDDPM. Then, the methods for finding a measured output $H(x)$, which guarantee the solvability of the DDDPM, were suggested. Future work will include improving the formula for computing the controlled invariant function ξ such that one does not need to assume that controlled output y_* has vector relative degree.

ACKNOWLEDGEMENTS

The work of A. Kaldmäe and Ü. Kotta was supported by grant PUT481 of the Estonian Research Council. The work of A. Shumsky and A. Zhirabok was supported by the Russian Scientific Foundation (project 16-19-00046). The publication costs of this article were covered by the Estonian Academy of Sciences.

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Häiringu kompenseerimine mõõdetava tagasisidega: sensorite paiknemine

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On uuritud võimalusi sensorite paigutamiseks nii, et häiringu kompenseerimise ülesanne oleks lahenduv dünaamilise mõõdetavatest väljunditest sõltuva tagasiside abil. Põhitulemused on esitatud diskreetse ajaga mittelineaarsete süsteemide klassile ja seejärel on käsitletud pideva ajaga süsteeme, keskendudes ainult erinevustele diskreetse juhuga võrreldes. Esiteks: töös on täiustatud tulemusi häiringu kompenseerimiseks. Kui varasemalt sõltusid tingimused häiringu kompenseerimiseks mõõdetava tagasisidega teatud invariantsest vektorfunktsioonist, siis antud artiklis on kahel loomulikul eeldusel tõestatud valem selle invariantse vektorfunktsiooni arvutamiseks. Järgnevalt on mõõdetava väljundi leidmiseks välja pakutud mitmeid meetodeid, mis garanteerivad häiringu kompenseerimise ülesande lahenduvuse. See mõõdetav väljund määrab sensorite arvu ja koha, kuhu need tuleb paigutada. Välja pakutud meetoditega otsitakse minimaalse dimensiooniga mõõdetavat väljundit, mis vastab minimaalsele sensorite arvule. Artiklis esitatud tulemusi on demonstreeritud mitme näitega.