Proc. Estonian Acad. Sci. Phys. Math., 2006, **55**, 1, 58–67

Dynamic response of an elevator car due to stochastic rail excitation

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Received 17 October 2005

Abstract. Elevator cars suffer from vibrations caused by irregularities in the guide system. These irregularities include random imperfections that are responsible for excitation of stochastic nature, resulting in the stochastic response of the car. The present paper focuses on this aspect of the car vibration. The nonstationary equations for the second-order statistical moments are formulated and a model example using the parameters of a typical building elevator installation is presented. It is shown that the weaker the correlation of the rail excitation process the higher the variance of the dynamic displacement of the car.

Key words: elevator car, suspension rope, stochastic excitation, vibration, stochastic process, variance.

1. INTRODUCTION

Lift cars are subjected to vibrations caused by various sources of excitation. They include lateral excitations due to the irregularities and imperfections of the rail guide system resulting from the accumulation of manufacturing errors [1,2]. Lateral vibrations of the car are then transmitted to the suspension and compensating ropes resulting in adverse dynamic behaviour of the entire elevator system. In particular, excessive friction wear affects the integrity of the suspension system and reduces the safe service life of the installation [3,4].

In general, the nature of guide rail imperfections should be classified as *nondeterministic*. If the unevenness of guide rails is measured, then the record for one rail will be different from that for the other. Thus, these imperfections are *random* or *stochastic* [5]. Consequently, the response of the car–hoist rope system is also a random phenomenon. Thus, the excitation caused by guide rail imperfections could be described by a *stochastic process* so that the methods of stochastic dynamics could be employed to seek a description of the dynamic behaviour of the lift car–suspension rope system in terms of *probabilities*. However, the stochastic analysis is complicated because the dynamic characteristics of the suspension ropes are time-variant. Variation of their length results in the change of the mass, stiffness, and damping characteristics of the system. Consequently, slow variation of the natural frequencies occurs, rendering the entire system nonstationary [6,7].

This paper is based on the results of a fundamental study to predict the car vibrations induced by the stochastic guide rail excitation, focusing on the presentation of the necessary theory and analysis of relevant models. The model applied in the analysis takes into account the fact that the dynamic characteristics of the suspension ropes are of time-variant nature. The guide rail excitation mechanism is subsequently discussed and a case study is presented where the stochastic response of the lift car is determined and analysed.

2. DYNAMIC MODEL OF THE CAR–SUSPENSION ROPE SYSTEM

The configuration of a typical traction elevator installation is shown in Fig. 1. The main components of a modern traction elevator system are the driving machine with traction sheave, elevator car assembly which carries passengers, the counterweight for balancing the weight of the car assembly and a portion of the rated load, the suspension means for the car and counterweight which are typically steel wire ropes, ropes for compensating tensile forces over the traction sheave, and travelling cables supplying the elevator car with electrical energy and connecting the car with the control unit. A diverting pulley may also be used to deflect the suspension rope to ensure that sufficient distance between the car and counterweight centre lines is maintained.

In order to predict correctly the vibration interaction in a lift installation, the car–suspension rope system should be considered using an adequate dynamic model. Assuming that the lift drive control system allows realization of an accurately prescribed motion of the traction sheave, the car and counterweight sides can be considered separately in the dynamic model. A schematic illustration of the model is shown in Fig. 2. In this representation the car assembly of mass M is suspended by the hoist rope of mass m per unit length and the car–guide rail interface is represented by a linear spring of the equivalent stiffness coefficient k. The kinematic excitation, caused by unevenness and bending of the guide rails, is modelled by the base motion $s(l)$, where $l(t) = l(0) \pm \int_0^t V(\xi)\,\mathrm{d}\xi$ represents the distance travelled by the lift at speed V. The signs "+" and "–" correspond to descending and ascending, respectively. The parameter $L = L(t)$ denotes the time-varying length of the rope. The variation of

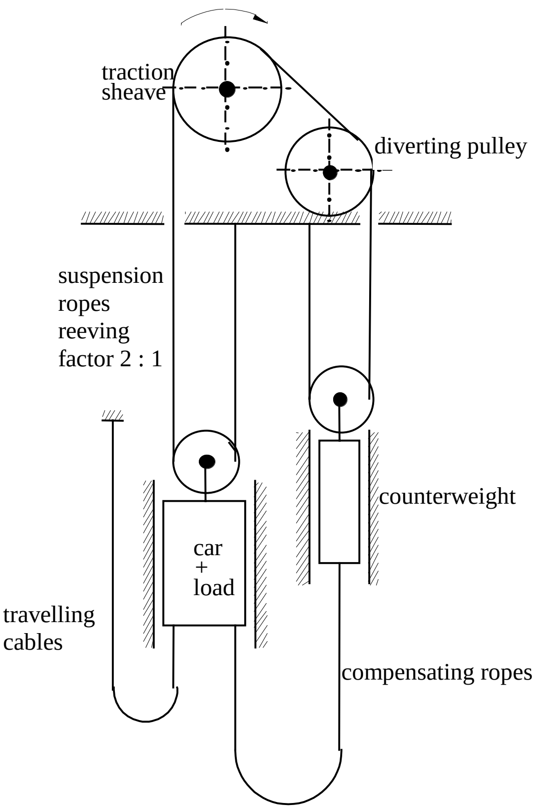

Fig. 1. Typical configuration of an elevator installation.

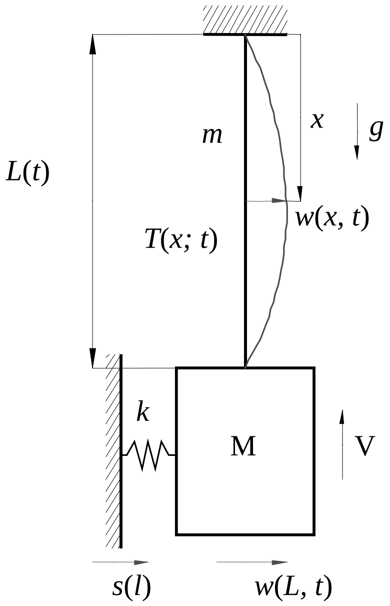

Fig. 2. Model of a lift car–hoist rope system.

the rope length is assumed to be *slow* and can be considered a function of slow time defined as $\tau = \varepsilon t$, where $\varepsilon << 1$ represents a small parameter given as

$$\varepsilon = \frac{V}{\omega_0 L_0}, \tag{1}$$

where ω_0 denotes the lowest natural frequency and L_0 is the corresponding length of the rope [7].

3. EQUATIONS OF MOTION

Noting that the speed of the rope is $\dot{L} = \varepsilon \, \mathrm{d}L/\mathrm{d}\tau$, where the overdot indicates total differentiation with respect to time t, the lateral oscillations $w(x,t)$ of the rope are described by the following equation:

$$m(w_{tt} + 2\varepsilon L' w_{xt} + \varepsilon^2 L'^2 w_{xx} + \varepsilon^2 L'' w_x) + \gamma(w_t + \varepsilon L' w_x) - (T w_x)_x = 0, \qquad (2)$$

where x is spatial coordinate measured as shown in Fig. 2, $(\,)'$ denotes differentiation with respect to slow time τ, $(\,)_t$ designates partial derivative with respect to time t, $(\,)_x$ denotes partial derivative with respect to x, γ is the constant damping parameter, and T is the mean tension of the rope. The boundary condition at the sheave end $x = 0$ is assumed to be trivial and the equation of motion of the car is defined by the boundary condition at $x = L$, given as

$$M(w_{tt} + 2\varepsilon L' w_{xt} + \varepsilon^2 L'^2 w_{xx} + \varepsilon^2 L'' w_x)|_{x=L} + k(w - s)|_{x=L} + (T w_x)|_{x=L} = 0. \qquad (3)$$

An approximate solution to Eq. (2) with the boundary condition (3) is sought using the expansion

$$w(x, t) = \sum_{n=1}^{N} \Psi_n(x; \tau)\, q_n(t), \qquad (4)$$

where

$$\Psi_n(x; \tau) = \sin \beta_n(\tau) x \qquad (5)$$

represent the slowly varying natural vibration modes (eigenfunctions) of the rope–car system with uniformly distributed slowly varying mean tension, expressed as $T(\tau) = [M + \frac{1}{2} mL(\tau)]g$. The slowly varying eigenvalues $\beta_n(\tau)$ are defined by the frequency equation

$$\left(k - \frac{M}{m} T(\tau)\beta_n^2\right) \sin \beta_n L(\tau) + T(\tau)\beta_n \cos \beta_n L(\tau) = 0. \qquad (6)$$

It should be noted that the eigenvalues β_n are related to the natural frequencies ω_n by $\beta_n = \omega_n / \bar{c}$, and $\bar{c} = (T/m)^{1/2}$ represents the speed of the lateral wave.

By substituting Eq. (4) into the equation of motion (2) and into the boundary condition (3), and orthogonalizing the result with respect to the natural modes, then neglecting the terms $O(\varepsilon)$ and $O(\varepsilon^2)$, the following system of differential equations, representing the dynamic behaviour of the car–rope system is obtained:

$$\ddot{q}_r(t) + 2\varsigma_r \omega_r(\tau)\dot{q}_r + \omega_r^2(\tau) q_r(t) = \frac{\Psi_r(L(\tau))}{m_r(\tau)} ks(t), \quad r = 1, 2, \ldots, N, \qquad (7)$$

where $m_r(\tau) = m\int_0^L \Psi_r^2(x; \tau)\,\mathrm{d}x + M\Psi_r^2(L)$ and the coefficients ς_r represent modal damping in the rope.

4. THE STOCHASTIC PROBLEM

In the dynamic analysis of road vehicles the random road profile is usually idealized as a zero-mean stationary Gaussian process. The same assumption about the nature of the rail imperfections is adopted in the following analysis.

Hence, the rail profile is assumed to be represented as a zero-mean stationary Gaussian process $s(l)$, characterized by the autocorrelation function

$$K_s(\lambda) = \sigma_s^2 e^{-\alpha|\lambda|}, \tag{8}$$

where σ_s^2 represents the variance of the random process $s(l)$, $\lambda = l_2 - l_1$ $(l_i = Vt_i,\ i = 1, 2)$, and α is the correlation parameter. If the correlation between the values $s(l_2)$ and $s(l_1)$ of the random process $s(l)$ is strong, α is small, and if this correlation is weak, α is large.

The corresponding spectral density function of the power is given as

$$S_s(\omega) = \frac{\sigma_s^2 \alpha}{\pi(\alpha^2 + \omega^2)}. \tag{9}$$

This represents a typical spectral density of many wide-band stationary stochastic processes. The stochastic process $s(l)$, characterized by the autocorrelation function (8) and the spectral density (9), is governed by Itô’s stochastic differential equation

$$\mathrm{d}s = -\alpha s \mathrm{d}l + \sigma_s \sqrt{2\alpha}\,\mathrm{d}W(l), \tag{10}$$

where $W(l)$ is the standard Wiener (Brownian motion) process. If the velocity of the car $V = \mathrm{d}l/\mathrm{d}t$ is constant, then we obtain Itô’s stochastic differential equation in the following form:

$$\mathrm{d}s = -\alpha s V \mathrm{d}t + \sigma_s \sqrt{2\alpha V}\,\mathrm{d}W(t). \tag{11}$$

The equations governing the state variables $Z_i(t)$, $i = 1, 2, \ldots, 2N+1$, are then as follows:

$$\mathrm{d}\mathbf{Z}(t) = \mathbf{C}(L(\tau))\mathbf{Z}(t)\mathrm{d}t + \mathbf{b}\mathrm{d}W(t). \tag{12}$$

Assuming that the response of the system is dominated by one mode, the expansion (4) may be confined to a single mode. Consequently, the single-mode approximation yields

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega_r^2 & -2\varsigma_r\omega_r & \dfrac{\Psi_r(L(\tau))}{m_r(\tau)}k \\ 0 & 0 & -\alpha V \end{bmatrix}, \mathbf{Z} = [q_r \quad \dot{q}_r \quad s]^T, \mathbf{b} = [0 \quad 0 \quad \sigma_s\sqrt{2\alpha V}\,]^T. \tag{13}$$

Differential equations governing directly the statistical moments of the response may be obtained from Itô's differential rule [8], which has the following form for the above problem:

$$\mathrm{d}\Phi(t,\mathbf{Z}(t))=\frac{\partial\Phi(t,\mathbf{Z}(t))}{\partial t}\mathrm{d}t+\sum_j\frac{\partial\Phi(t,\mathbf{Z}(t))}{\partial Z_j}c_j(t,\mathbf{Z}(t))\,\mathrm{d}t$$

$$+\frac{1}{2}\sum_{j,k}\frac{\partial^2\Phi(t,\mathbf{Z}(t))}{\partial Z_j Z_k}b_j b_k\,\mathrm{d}t+\sum_j\frac{\partial\Phi(t,\mathbf{Z}(t))}{\partial Z_j}b_j\mathrm{d}W(t), \tag{14}$$

where $c_j(t,\mathbf{Z}(t))$ represents the drift term of Eq. (12).

By assuming that $\Phi(t,\mathbf{Z}(t))=Z_i(t)Z_j(t),$ and performing the expectation operation over the rule (14) and interchanging the order of expectation and differentiation with respect to time, the differential equations for the statistical moments of the second-order response $\kappa_{ij}=E[Z_i(t)Z_j(t)]$ are obtained as

$$\frac{\mathrm{d}\kappa_{ij}}{\mathrm{d}t}=2\{C_{im}\kappa_{mj}\}_s+b_ib_j;\quad i,\ j=1,2,\dots,2N+1, \tag{15}$$

where $\{\ldots\}_s$ denotes the symmetrization operation

$$2\{C_{im}\kappa_{mj}\}_s=2\frac{\sum_{m=1}^{2N+1}(C_{im}\kappa_{mj}+C_{jm}\kappa_{mi})}{2}. \tag{16}$$

The variance of the lateral displacement $w(x,t)$ is subsequently expressed as

$$\sigma_w^2(x,t)=\sum_{i=1}^{N}\sum_{j=1}^{N}\kappa_{ij}(t)\Psi_i(x)\Psi_j(x). \tag{17}$$

5. SIMULATION AND RESULTS

The model described above is used to conduct a simulation study of the random response of a car–suspension rope system. The suspension is a $2:1$ roping system with six 12 mm ropes of mass per unit length $m=0.65$ kg/m each. The simulation involves the ascending and descending of a fully loaded car of mass 2000 kg with rated load of 1250 kg (so that $M=3250$ kg) at two different speeds. The well height of the installation is 70 m, the car height is 3.2 m and the total travel height is 60 m. The damping ratio of 0.4 for the fundamental mode $(r=1)$ of the suspension rope system is assumed. The stiffness coefficient for the car guide shoe–rail interface is $k=2083$ N/m and the maximum roughness (deviation from the straight line) of the rail profile is assumed as $s_{\max}=1$ mm [1]. The standard deviation σ_s is subsequently assumed as the root mean square (RMS) value $\sigma_s=s_{\mathrm{RMS}}\approx 0.707 s_{\max}$.

The results of the simulation are presented in Figs 3–6. They demonstrate the fundamental features of the dynamic response of the lift car–hoist rope system subjected to the stochastic guide rail excitation represented as a zero-mean stationary Gaussian stochastic process. It is evident that the variance of the car response changes during the elevator travel. It appears that the higher the value of the coefficient α (thus, the weaker the correlation of the rail excitation process) the higher the variance of the dynamic displacement of the car. During ascent the variance is initially increasing rapidly with decreasing length of the suspension rope, and it starts decreasing slowly after reaching its maximum value, as demonstrated in Figs 3 and 4. This trend is more evident for the higher values of α (the less correlated excitation processes). On the other hand, during descent the behaviour of the variance is more regular: it is increasing with increasing length of the suspension rope (Figs 5 and 6). The variance of the car response depends on the rated speed of the elevator. It can be observed that the higher the speed the lower the variance of the car displacement during the entire ascending cycle. However, the behaviour during the descent appears to be more diverse: the variance is initially higher for lower speeds but from a certain height this trend becomes inverted (the higher the speed the larger the variance).

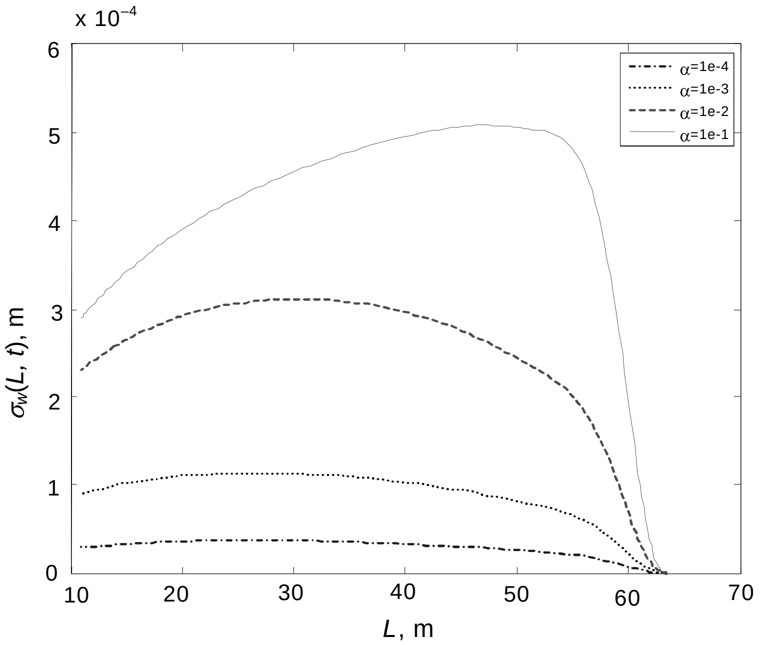

Fig. 3. Standard deviation of car displacement $\sigma_w(L,t)$ vs. rope length L; ascent at the rated speed $V = 1.5$ m/s.

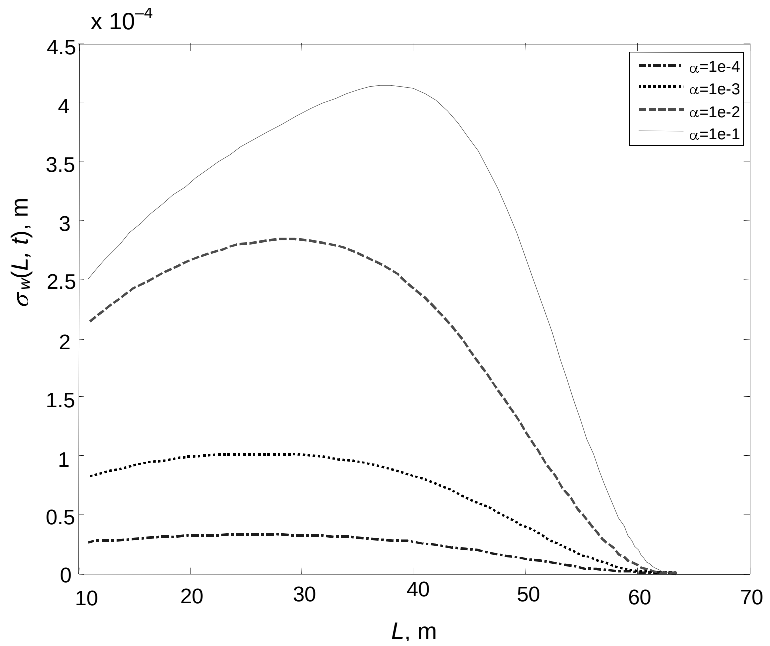

Fig. 4. Standard deviation of car displacement $\sigma_w(L,t)$ vs. rope length L; ascent at the rated speed $V = 6$ m/s.

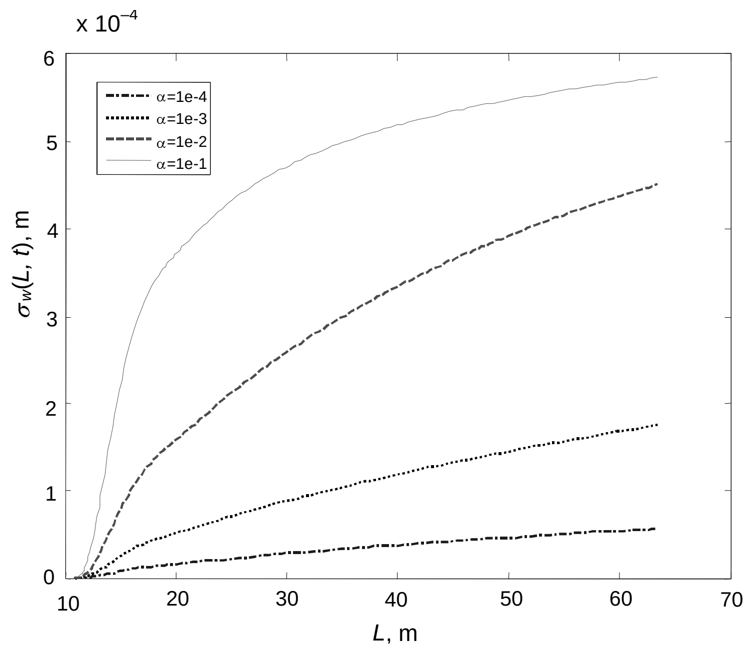

Fig. 5. Standard deviation of car displacement $\sigma_w(L,t)$ vs. rope length L; descent at the rated speed $V = 1.5$ m/s.

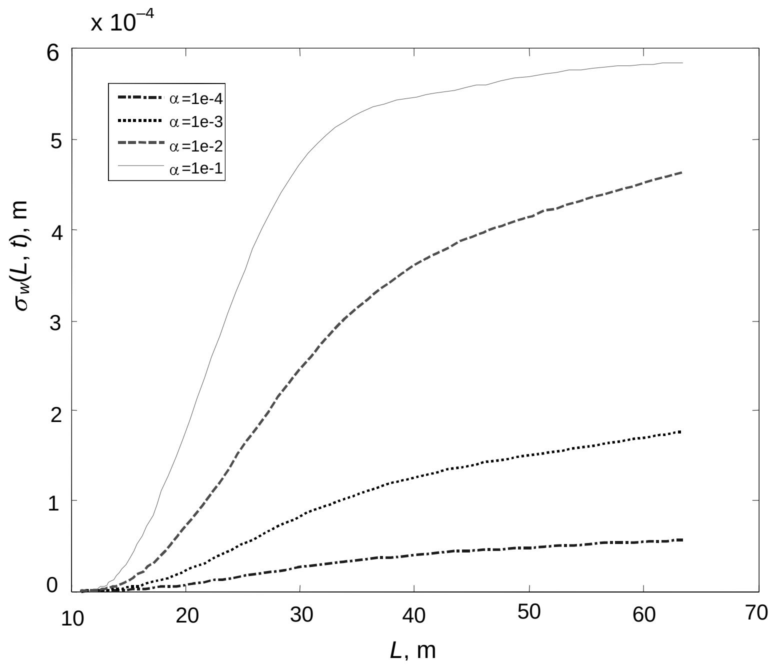

Fig. 6. Standard deviation of car displacement $\sigma_w(L,t)$ vs. rope length L; descent at the rated speed V = 6 m/s.

6. CONCLUSIONS

Poor ride quality of an elevator car is primarily caused by guide rail imperfections. Uneven bent rails, incorrect installation, and rough surfaces cause vibration of the car and of the suspension members. The rail excitation mechanism can be represented as a stochastic process and used in the dynamic model to predict the probabilistic characteristics of the car response.

The variation of the length of the ropes during the lift motion results in slow variation of the mass and stiffness of the entire system. This nonstationary nature of the elevator installation must be taken into account in the analysis of its stochastic response. The model applied in this paper accommodates this fundamental feature, and the results of numerical simulation demonstrate that the variance of the car response changes during the elevator travel. In particular, it is demonstrated that the weaker the correlation of the rail excitation process the higher the variance of the dynamic displacement of the car for the range of parameters used in the study. Also, the influence of the speed on the random response characteristics is apparent. It is shown that during the ascent the higher the speed the lower the variance of the car response. The influence of the speed during the descent is less regular. Closer examination of the simulation results reveals that the variance is initially higher for lower speeds, but from a certain travel height this trend becomes inverted.

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Tõstukikabiini dünaamiline reaktsioon juhtrööpa juhuslikele mõjutustele

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Juhtsüsteemi ebaregulaarsused kutsuvad esile tõstukikabiini võnkumise. Need on tingitud konstruktsiooni juhuslikest ebatäpsustest ja nende mõju on stohhastilise iseloomuga. Artiklis on vaadeldud kabiini võnkumisi stohhastilisest aspektist. On formuleeritud statistiliste momentide teist järku võrrandid ja analüüsitud tulemusi näidismudelil, mille parameetrid vastavad tüüpilisele ehitustõstukile. On näidatud, et mida nõrgem on korrelatsioon juhtrööpa mõjuga, seda suurem on kabiini dünaamilise paigutuse dispersioon.